Computational Appendix
to
"International Business Cycles with Complete Markets"

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Abstract

This appendix presents the derivations of the optimality conditions and describes how the benchmark model is solved numerically with the Parameterized Expectations Approach.

1 The Optimality Conditions

An equilibrium allocation in this economy can be computed as the solution to a social planner's problem. Taking the initial conditions $\{k_j(s_0), h_j(s_0), z_j(s_0)\}_{j\in J}$ as given, the planner chooses state-contingent plans $\{c_j(s^t), i_j(s^t), n_j(s^t)\}_{t=0,s^t\in S^t}^{\infty}$ for each agent $j \in J = \{H, F\}$ to maximize the expected discounted sum of their weighted utilities

$$\sum_{t=0}^{\infty}\beta^t \sum_{s^t\in S^t}\pi(s^t)\sum_{j=J}\omega_j u(c_j(s^t), h_j(s^{t-1}), n_j(s^t)), \tag{1}$$

subject to capital accumulation equation

$$k(s^t) = (1-\delta)k(s^{t-1}) + \phi\left(\frac{i(s^t)}{k(s^{t-1})}\right)k(s^{t-1}), \text{ for } j = J, \tag{2}$$

the law of motion for habits

$$h_j(s^t) = c_j(s^t), \text{ for } j = J, \tag{3}$$

as well as the global resource constraint

$$\sum_{j\in J}c_j(s^t) + \sum_{j\in J}i_j(s^t) = \sum_{j\in J}z_j(s^t)f(k_j(s^{t-1}), n_j(s^t)). \tag{4}$$

The Lagrangian associated with the planner's problem is given by

$$\begin{aligned}
L &= \sum_{t=0}^{\infty}\sum_{s^t\in S^t}\Bigg[\beta^t\pi\left(s^t\right)\sum_{j=J}\omega_j u(c_j\left(s^t\right),h_j\left(s^{t-1}\right),n_j\left(s^t\right)) \\
&\quad -\sum_{j\in J} m_j\left(s^t\right)\left(k\left(s^t\right)-(1-\delta)k\left(s^{t-1}\right)-\phi\left(\frac{i\left(s^t\right)}{k\left(s^{t-1}\right)}\right)k\left(s^{t-1}\right)\right) \\
&\quad -\sum_{j\in J} n_j\left(s^t\right)\left(h_j\left(s^t\right)-c_j\left(s^t\right)\right) \\
&\quad -\gamma\left(s^t\right)\left(\sum_{j\in J}c_j\left(s^t\right)+\sum_{j\in J}i_j\left(s^t\right)-\sum_{j\in J}z_j\left(s^t\right)f(k_j\left(s^{t-1}\right),n_j\left(s^t\right))\right)\Bigg],
\end{aligned}$$

where $\{m_j\left(s^t\right)\}_{t=0}^{\infty}$, $\left\{n_j\left(s^t\right)\right\}_{t=0}^{\infty}$ and $\left\{\gamma\left(s^t\right)\right\}_{t=0}^{\infty}$ are the state-contingent paths of Lagrange multipliers associated with constraints (2), (3), and (4). Equating the gradient of the Lagrangian to zero we obtain

$$\beta^t\pi\left(s^t\right)\omega_j u_1(c_j\left(s^t\right),h_j\left(s^{t-1}\right),n_j\left(s^t\right))+n_j\left(s^t\right)=\gamma\left(s^t\right), \tag{5}$$

$$m_j\left(s^t\right)\phi'\left(\frac{i_j\left(s^t\right)}{k_j\left(s^{t-1}\right)}\right)=\gamma\left(s^t\right), \tag{6}$$

$$\beta^t\pi\left(s^t\right)\omega_j u_3(c_j\left(s^t\right),h_j\left(s^{t-1}\right),n_j\left(s^t\right))+\gamma\left(s^t\right)z_j\left(s^t\right)f_2(k_j\left(s^{t-1}\right),n_j\left(s^t\right))=0, \tag{7}$$

$$\begin{aligned}
m_j\left(s^t\right) &= \sum_{s_{t+1}\in S} m_j\left(s^t,s_{t+1}\right)\left(1-\delta+\phi\left(\frac{i_j\left(s^t,s_{t+1}\right)}{k_j\left(s^t\right)}\right)-\phi'\left(\frac{i_j\left(s^t,s_{t+1}\right)}{k_j\left(s^t\right)}\right)\frac{i_j\left(s^t,s_{t+1}\right)}{k_j\left(s^t\right)}\right) \\
&\quad +\sum_{s_{t+1}\in S}\gamma\left(s^t,s_{t+1}\right)z_j\left(s^t,s_{t+1}\right)f_1(k_j\left(s^t,s_{t+1}\right),n_j\left(s^t,s_{t+1}\right)),
\end{aligned} \tag{8}$$

$$n_j\left(s^t\right)=\sum_{s_{t+1}\in S}\beta^{t+1}\pi\left(s^t,s_{t+1}\right)\omega_j u_2\left(c_j\left(s^t,s_{t+1}\right),h_j\left(s^t,s_{t+1}\right),n_j\left(s^t,s_{t+1}\right)\right), \tag{9}$$

where $u_1(\cdot)$ is the partial derivative of u with respect to its first argument. We use the same notation to denote the other partial derivatives.

The intertemporal conditions (8) and (9) can be rearranged as

$$\frac{n_j\left(s^t\right)}{\beta^t\pi\left(s^t\right)}=\sum_{s_{t+1}\in S}\frac{\beta^{t+1}\pi\left(s^t,s_{t+1}\right)}{\beta^t\pi\left(s^t\right)}\omega_j u_2(c_j\left(s^t,s_{t+1}\right),h_j\left(s^t,s_{t+1}\right),n_j\left(s^t,s_{t+1}\right)), \tag{10}$$

and

$$\begin{aligned}
\frac{m_j\left(s^t\right)}{\beta^t\pi\left(s^t\right)} &= \sum_{s_{t+1}\in S}\frac{m_j\left(s^t,s_{t+1}\right)}{\beta^{t+1}\pi\left(s^t,s_{t+1}\right)}\frac{\beta^{t+1}\pi\left(s^t,s_{t+1}\right)}{\beta^t\pi\left(s^t\right)} \\
&\quad\times\left(1-\delta+\phi\left(\frac{i_j\left(s^t,s_{t+1}\right)}{k_j\left(s^t\right)}\right)-\phi'\left(\frac{i_j\left(s^t,s_{t+1}\right)}{k_j\left(s^t\right)}\right)\frac{i_j\left(s^t,s_{t+1}\right)}{k_j\left(s^t\right)}\right) \\
&\quad+\sum_{s_{t+1}\in S}\frac{\gamma\left(s^t,s_{t+1}\right)}{\beta^{t+1}\pi\left(s^t,s_{t+1}\right)}\frac{\beta^{t+1}\pi\left(s^t,s_{t+1}\right)}{\beta^t\pi\left(s^t\right)}z_j\left(s^t,s_{t+1}\right)f_1(k_j\left(s^t,s_{t+1}\right),n_j\left(s^t,s_{t+1}\right))
\end{aligned} \tag{11}$$

By denoting $\widetilde{n}_j\left(s^t\right)=\frac{n_j\left(s^t\right)}{\beta^t\pi\left(s^t\right)}$, $\widetilde{m}_j\left(s^t\right)=\frac{m_j\left(s^t\right)}{\beta^t\pi\left(s^t\right)}$, and $\widetilde{\gamma}\left(s^t\right)=\frac{\gamma\left(s^t\right)}{\beta^t\pi\left(s^t\right)}$ we can rewrite equations (10) and (11) as

$$\widetilde{n}_j\left(s^t\right)=\beta\sum_{s_{t+1}\in S}\pi\left(s_{t+1}\mid s^t\right)\omega_j u_2(c_j\left(s^t,s_{t+1}\right),h_j\left(s^t,s_{t+1}\right),n_j\left(s^t,s_{t+1}\right)), \tag{12}$$

and

$$\begin{aligned}\widetilde{m}_j\left(s^t\right) &= \beta\sum_{s_{t+1}\in S}\pi\left(s_{t+1}\mid s^t\right)\widetilde{m}_j\left(s^t,s_{t+1}\right)\\ &\quad\times\left(1-\delta+\phi\left(\frac{i_j\left(s^t,s_{t+1}\right)}{k_j\left(s^t\right)}\right)-\phi'\left(\frac{i_j\left(s^t,s_{t+1}\right)}{k_j\left(s^t\right)}\right)\frac{i_j\left(s^t,s_{t+1}\right)}{k_j\left(s^t\right)}\right)\\ &\quad+\beta\sum_{s_{t+1}\in S}\pi\left(s_{t+1}\mid s^t\right)\widetilde{\gamma}\left(s^t,s_{t+1}\right)z_j\left(s^t,s_{t+1}\right)f_1(k_j\left(s^t,s_{t+1}\right),n_j\left(s^t,s_{t+1}\right)).\end{aligned}$$

In a similar way, equations (5) and (7) can be re-written as

$$\omega_j u_1(c_j\left(s^t\right),h_j\left(s^{t-1}\right),n_j\left(s^t\right))+\widetilde{n}_j\left(s^t\right)=\widetilde{\gamma}\left(s^t\right), \tag{13}$$

and

$$\omega_j u_3(c_j\left(s^t\right),h_j\left(s^{t-1}\right),n_j\left(s^t\right))+\widetilde{\gamma}\left(s^t\right)z_j\left(s^t,s_{t+1}\right)f_2(k_j\left(s^t,s_{t+1}\right),n_j\left(s^t,s_{t+1}\right))=0. \tag{14}$$

Let $\Lambda_j\left(s^t\right)$ denote the marginal utility of consumption of agent j after history s^t. Then from (12) and (13) it follows that

$$\Lambda_j\left(s^t\right)=u_1(c_j\left(s^t\right),h_j\left(s^{t-1}\right),n_j\left(s^t\right))+\beta\sum_{s_{t+1}\in S}\pi\left(s_{t+1}\mid s^t\right)u_2(c_j\left(s^t,s_{t+1}\right),h_j\left(s^t,s_{t+1}\right),n_j\left(s^t,s_{t+1}\right)),$$

where $\pi\left(s_{t+1}\mid s^t\right)$ denotes the conditional probability of s_{t+1} given s^t, and $\pi\left(s^t\mid s^t\right)=1$.

Let $R_j\left(s^t,s_{t+1}\right)$ denote the realized one-period gross rate of return on capital in country j after realization of history $\left(s^t,s_{t+1}\right)$

$$\begin{aligned}R_j\left(s^t,s_{t+1}\right) &= \phi'\left(\frac{i_j\left(s^t,s_{t+1}\right)}{k_j\left(s^t\right)}\right)z_j\left(s^t,s_{t+1}\right)f_1(k_j\left(s^t,s_{t+1}\right),n_j\left(s^t,s_{t+1}\right))\\ &\quad+\left(1-\delta+\phi\left(\frac{i_j\left(s^t,s_{t+1}\right)}{k_j\left(s^t\right)}\right)-\phi'\left(\frac{i_j\left(s^t,s_{t+1}\right)}{k_j\left(s^t\right)}\right)\frac{i_j\left(s^t,s_{t+1}\right)}{k_j\left(s^t\right)}\right)\\ &\quad\times\phi'\left(\frac{i_j\left(s^t\right)}{k_j\left(s^{t-1}\right)}\right)/\phi'\left(\frac{i_j\left(s^t,s_{t+1}\right)}{k_j\left(s^t\right)}\right).\end{aligned}$$

Then the first order conditions can be reformulated as

$$\Lambda_H\left(s^t\right)=\left(\frac{\omega_F}{\omega_H}\right)\Lambda_F\left(s^t\right), \tag{15}$$

$$\Lambda_j\left(s^t\right)=\beta\sum_{s_{t+1}\in S}\pi\left(s_{t+1}\mid s^t\right)\Lambda_j\left(s^t,s_{t+1}\right)R_j\left(s^t,s_{t+1}\right),\text{for } j\in J, \tag{16}$$

$$u_3(c_j\left(s^t\right),h_j\left(s^{t-1}\right),n_j\left(s^t\right))+\Lambda_j\left(s^t\right)z_j\left(s^t\right)f_2(k_j\left(s^{t-1}\right),n_j\left(s^t\right))=0,\ \text{for } j\in J. \tag{17}$$

2 Solving the Model with the Parameterized Expectation Approach

2.1 Optimality conditions with the functional forms

The instantaneous utility function takes the form

$$u(c,h,n)=\frac{1}{1-\sigma}\left(c-bh-\chi\frac{n^{1+\eta}}{1+\eta}\right)^{1-\sigma}.$$

The production function is

$$y=zf(k,n)=zk^{\alpha}n^{1-\alpha}.$$

The capital adjustment cost function is

$$\begin{aligned}\phi\left(x\right) &= \frac{a_1}{1-1/\xi}\left(x\right)^{1-1/\xi}+a_2,\\ \phi'\left(x\right) &= a_1x^{-1/\xi}=\left(\frac{\delta}{x}\right)^{1/\xi}\end{aligned}$$

where the restrictions that $\phi'\left(\delta\right)=1$ and $\phi\left(\delta\right)=\delta$ require that $a_1=\delta^{1/\xi}$ and $a_2=\frac{\delta}{1-\xi}$. Symmetry between the two economies implies that $\omega_H=\omega_F$. Incorporating specific functional forms, the optimality conditions can be rewritten as

$$\Lambda_H\left(s^t\right)=\Lambda_F\left(s^t\right),$$

$$\Lambda_j\left(s^t\right)=\beta E_t\left[\Lambda_j\left(s^t\right)R_j\left(s^t\right)\right],$$

$$\Lambda_j\left(s^t\right)=\left(c_j\left(s^t\right)-bh_j\left(s^{t-1}\right)-\chi\frac{n_j\left(s^t\right)^{1+\eta}}{1+\eta}\right)^{-\sigma}-b\beta E_t\left[\left(c_j(s^{t+1})-bh_j(s^t)-\chi\frac{n_j(s^{t+1})^{1+\eta}}{1+\eta}\right)^{-\sigma}\right],$$

$$\frac{\Lambda_j\left(s^t\right)}{\chi n_j\left(s^t\right)^{\eta}\left(c_j\left(s^t\right)-bh_j\left(s^{t-1}\right)-\chi\frac{n_j(s^t)^{1+\eta}}{1+\eta}\right)^{-\sigma}}=\frac{1}{\left(1-\alpha\right)z_j\left(s^t\right)k_j\left(s^{t-1}\right)^{\alpha}n_j\left(s^t\right)^{-\alpha}},$$

$$R_{t+1}=a_1\left(\frac{i_j(s^t)}{k_j(s^{t-1})}\right)^{-1/\xi}\left[\alpha\frac{y_j(s^{t+1})}{k_j(s^t)}+\left(\frac{i_j(s^{t+1})}{k_j(s^t)}\right)^{1/\xi}\left(\frac{1-\delta+a_2}{a_1}+\frac{1}{\xi-1}\left(\frac{i_j(s^{t+1})}{k_j(s^t)}\right)^{1-1/\xi}\right)\right],$$

$$c_H\left(s^t\right)+c_F\left(s^t\right)+i_H\left(s^t\right)+i_F\left(s^t\right)=y_H\left(s^t\right)+y_F\left(s^t\right).$$

2.2 Parameter Values for the Benchmark model

Productivity follows a process similar to that specified by (Kehoe and Perri, 2002)

$$\begin{bmatrix}\log(z_H\left(s^t\right))\\ \log(z_F\left(s^t\right))\end{bmatrix}=\begin{bmatrix}0.95 & 0\\ 0 & 0.95\end{bmatrix}\begin{bmatrix}\log(z_H\left(s^{t-1}\right))\\ \log(z_F\left(s^{t-1}\right))\end{bmatrix}+\begin{bmatrix}\varepsilon_H\left(s^t\right)\\ \varepsilon_F\left(s^t\right)\end{bmatrix}.$$

The innovations to the productivity process are zero mean serially independent bivariate normal random variables with the contemporaneous covariance matrix

$$E\left[\varepsilon_t\varepsilon_t'\right]=0.007^2\cdot\begin{bmatrix}1 & 0.25\\ 0.25 & 1\end{bmatrix}.$$

Standard/estimated values are as follows:

- Capital income share $\alpha = 0.36$ and coefficient of relative risk aversion $\sigma = 2$, as in (Kehoe and Perri, 2002);
- Elasticity of labor supply $1/\eta = 1.43$ -> $\eta = 1/1.43 = 0.6993$, as in Correia et al. (1995);
- Intensity of habits $b = 0.73$, as in Jermann (1998).

The calibration targets are: $n_{ss} = 1/3; i_{ss}/y_{ss} = 0.25; k_{ss}/y_{ss} = 10$. The calibrated parameters are as follows:

- Depreciation rate: $\delta = i_{ss}/k_{ss} = (i_{ss}/y_{ss}) / (k_{ss}/y_{ss}) = 0.025$;
- Discount factor: $\beta = (\alpha (y_{ss}/k_{ss}) + 1 - \delta)^{-1} = (0.36 \cdot 0.1 + 1 - 0.025)^{-1} = 0.989$;
- From $1 = \beta R_{ss} = \beta \left(\alpha k_{ss}^{\alpha-1} n_{ss}^{1-\alpha} + 1 - \delta\right)$ it follows that

$$k_{ss} = \left(\frac{\alpha}{1/\beta - 1 + \delta}\right)^{\frac{1}{1-\alpha}} n_{ss} = \left(\frac{0.36}{1/0.989 - 1 + 0.025}\right)^{\frac{1}{1-0.36}} (1/3) = 12.172;$$

- From the labor supply equation in the non-stochastic steady state $\chi n_{ss}^{\eta} = (1-b\beta)(1-\alpha) k_{ss}^{\alpha} n_{ss}^{-\alpha}$, it follows that the weight of labor in the utility function χ is:

$$\begin{aligned}\chi &= (1-b\beta)(1-\alpha)\frac{k_{ss}^{\alpha}}{n_{ss}^{\alpha+\eta}} = (1-b\beta)(1-\alpha)\left(\frac{\alpha}{1/\beta - 1 + \delta}\right)^{\frac{\alpha}{1-\alpha}} n_{ss}^{-\eta} \\ &= (1 - 0.73 \cdot 0.989)(1-0.36)\left(\frac{0.36}{1/0.989 - 1 + 0.025}\right)^{\frac{0.36}{1-0.36}} (1/3)^{-(1/1.43)} = 1.401.\end{aligned}$$

The other steady-state values are as follows:

$$i_{ss} = \delta k_{ss} = 0.025 \cdot 12.108 = 0.3043;$$
$$y_{ss} = k_{ss}^{\alpha} n_{ss}^{1-\alpha} = (12.172)^{0.36} (1/3)^{(1-0.36)} = 1.2172;$$
$$c_{ss} = y_{ss} - i_{ss} = 1.2172 - 0.3043 = 0.9129.$$

2.3 The Numerical Procedure

The model is solved using the Parameterized Expectations Approach of den Haan and Marcet (1990). The approach is to replace the conditional expectations in (16) and (17) with smooth parametric approximation functions of the current state variables and a vector of parameters, and then iterate on the parameter values until a rational expectations equilibrium is achieved. The four conditional expectations are parameterized as follows

$$E_t\left[\left(c_{Ht+1} - bc_{Ht} - \chi\frac{n_{Ht+1}^{1+\eta}}{1+\eta}\right)^{-\sigma}\right] = \Psi(\omega_1; \mathbf{x}_t)$$

$$E_t\left[\left(c_{Ft+1}-bc_{Ft}-\chi\frac{n_{Ft+1}^{1+\eta}}{1+\eta}\right)^{-\sigma}\right]=\Psi(\omega_2;\mathbf{x}_t)$$

$$\Lambda_{Ht}=\beta E_t\left[\Lambda_{Ht+1}R_{Ht+1}\right]=\Psi(\omega_3;\mathbf{x}_t)$$

$$k_{Ft+1}=\beta E_t\left[\Lambda_{Ft+1}R_{Ft+1}k_{Ft+1}\right]=\Psi(\omega_4;\mathbf{x}_t)$$

where $\mathbf{x}_t=[k_{Ht},k_{Ft},c_{Ht-1},c_{Ft-1},z_{Ht},z_{Ft}]$. From the first order condition for consumption in the home country we have

$$\Lambda_{Ht}=\left(c_{Ht}-bc_{Ht-1}-\chi\frac{n_{Ht}^{1+\eta}}{1+\eta}\right)^{-\sigma}-b\beta E_t\left[\left(c_{Ht+1}-bc_{Ht}-\chi\frac{n_{Ht+1}^{1+\eta}}{1+\eta}\right)^{-\sigma}\right].$$

Re-arranging yields

$$\left(c_{Ht}-bc_{Ht-1}-\chi\frac{n_{Ht}^{1+\eta}}{1+\eta}\right)^{-\sigma}=\Lambda_{Ht}+b\beta E_t\left[\left(c_{Ht+1}-bc_{Ht}-\chi\frac{n_{Ht+1}^{1+\eta}}{1+\eta}\right)^{-\sigma}\right]=\Psi(\omega_3;\mathbf{x}_t)+b\beta\Psi(\omega_1;\mathbf{x}_t).$$

From the first order condition for labor in the home country

$$\frac{\chi n_{Ht}^{\eta}\left(c_{Ht}-bc_{Ht-1}-\chi\frac{n_{Ht}^{1+\eta}}{1+\eta}\right)^{-\sigma}}{\Lambda_{Ht}}=(1-\alpha)\,z_{Ht}k_{Ht}^{\alpha}n_{Ht}^{-\alpha}$$

it follows that

$$n_{Ht}^{\eta+\alpha}=\frac{(1-\alpha)\,z_{Ht}k_{Ht}^{\alpha}}{\chi}\frac{\Lambda_{Ht}}{\left(c_{Ht}-bc_{Ht-1}-\chi\frac{n_{Ht}^{1+\eta}}{1+\eta}\right)^{-\sigma}}=\frac{(1-\alpha)\,z_{Ht}k_{Ht}^{\alpha}}{\chi}\frac{\Psi(\omega_3;\mathbf{x}_t)}{\Psi(\omega_3;\mathbf{x}_t)+b\beta\Psi(\omega_1;\mathbf{x}_t)}.$$

From the risk-sharing condition:

$$\Lambda_{Ht}=\Lambda_{Ft}$$

we obtain

$$\left(c_{Ft}-bc_{Ft-1}-\chi\frac{n_{Ft}^{1+\eta}}{1+\eta}\right)^{-\sigma}=\Lambda_{Ft}+b\beta E_t\left[\left(c_{Ft+1}-bc_{Ft}-\chi\frac{n_{Ft+1}^{1+\eta}}{1+\eta}\right)^{-\sigma}\right]=\Psi(\omega_3;\mathbf{x}_t)+b\beta\Psi(\omega_2;\mathbf{x}_t),$$

and from the foreign labor supply equation we get

$$n_{Ft}^{\eta+\alpha}=\frac{(1-\alpha)\,z_{Ft}k_{Ft}^{\alpha}}{\chi}\frac{\Psi(\omega_3;\mathbf{x}_t)}{\Psi(\omega_3;\mathbf{x}_t)+b\beta\Psi(\omega_2;\mathbf{x}_t)}.$$

Current consumption in each country is therefore given by:

$$c_{Ht}=\left[\Psi(\omega_3;\mathbf{x}_t)+b\beta\Psi(\omega_1;\mathbf{x}_t)\right]^{-\frac{1}{\sigma}}+bc_{Ht-1}+\chi\frac{n_{Ht}^{1+\eta}}{1+\eta},\tag{18}$$

$$c_{Ft}=\left[\Psi(\omega_3;\mathbf{x}_t)+b\beta\Psi(\omega_2;\mathbf{x}_t)\right]^{-\frac{1}{\sigma}}+bc_{Ft-1}+\chi\frac{n_{Ft}^{1+\eta}}{1+\eta}.\tag{19}$$

Labor in each country is given by

$$n_{Ht}^{\eta+\alpha}=\frac{(1-\alpha)\,z_{Ht}k_{Ht}^{\alpha}}{\chi}\frac{\Psi(\omega_3;\mathbf{x}_t)}{\Psi(\omega_3;\mathbf{x}_t)+b\beta\Psi(\omega_1;\mathbf{x}_t)},\tag{20}$$

$$n_{Ft}^{\eta+\alpha} = \frac{(1-\alpha)\, z_{Ft} k_{Ft}^{\alpha}}{\chi} \frac{\Psi(\omega_3; \mathbf{x}_t)}{\Psi(\omega_3; \mathbf{x}_t) + b\beta\Psi(\omega_2; \mathbf{x}_t)}. \tag{21}$$

The algorithm is implemented as follows:[1]

1. Obtain an initial guess for $\boldsymbol{\omega} = [\omega_1, \omega_2, \omega_3, \omega_4]$. Fix $k_{j0} = k_{ss}$, $h_{j0} = c_{ss}$ and $z_{j0} = 1$ for $j \in J$, and draw a sample of size T of the exogenous stochastic shock $\{z_{Ht}, z_{Ft}\}_{t=0}^{T}$.
2. Replace the conditional expectations with the parameterized functions $\Psi(\omega_r; \mathbf{x}_t)$, $r = 1...4$. Calculate $\{n_{Ht}, n_{Ft}, c_{Ht}, c_{Ft}, h_{Ht}, h_{Ft}\}_{t=0}^{T}$ using equations (18), (19), (20) and (21), and the law of motion for habits (3). Calculate $\{y_{Ht}, y_{Ft}, i_{Ht}, i_{Ft}, k_{Ht}\}_{t=0}^{T}$ using the production function, the law of motion for capital (2) and the global resource constraint (4). Similarly compute $\{\Lambda_{Ht}, R_{Ht}, R_{Ft}\}_{t=0}^{T}$.
3. Set

$$\begin{aligned}
Y_t^1(\boldsymbol{\omega}) &\equiv \left(c_{Ht+1} - b c_{Ht} - \chi \frac{n_{Ht+1}^{1+\eta}}{1+\eta} \right)^{-\sigma}, \\
Y_t^2(\boldsymbol{\omega}) &\equiv \left(c_{Ft+1} - b c_{Ft} - \chi \frac{n_{Ft+1}^{1+\eta}}{1+\eta} \right)^{-\sigma}, \\
Y_t^3(\boldsymbol{\omega}) &\equiv \beta \left[\Lambda_{Ht+1} R_{Ht+1} \right], \\
Y_t^4(\boldsymbol{\omega}) &\equiv \Lambda_{Ft+1} R_{Ft+1} k_{Ft+1}.
\end{aligned}$$

and minimize the sum of squared residuals for the equation $Y_t^r(\boldsymbol{\omega}) = \Psi(\omega_r; \mathbf{x}_t(\boldsymbol{\omega})) + \nu_t^r$, $r = 1...4$, where ν_t^r is the regression error. That is, find

$$G(\omega_r) = \arg\min_{\zeta} \frac{1}{T} \sum_{t=0}^{T} ||Y_t^r(\boldsymbol{\omega}) - \Psi(\zeta; \mathbf{x}_t(\boldsymbol{\omega}))||^2$$

where ζ is the parameter vector to be estimated.

4. Iterating on w_r, find the fixed point $w_r^* = G(w_r^*)$. Update w_r using the algorithm $\omega_r(\tau+1) = (1-\mu)\omega_r(\tau) + \mu G(\omega_r(\tau))$ for $\mu > 0$, $\omega_r(0)$ given.

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[1]For a formal description of the PEA algorithm and related proofs, see Marcet and Marshall (1994).