The Cost of Adjustment: On Comovement between the Trade Balance and the Terms of Trade

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Abstract

The S-shaped cross-correlation function between the trade balance and the terms of trade has been documented for several countries and time frames. The ability of two-country, two-good business cycle models to reproduce this regularity hinges on the dynamics of capital formation. We consider the consequences of modeling the adjustment costs for comovements in the trade balance and the terms of trade. Both complete and incomlete markets models with capital adjustment costs *à la* Hayashi (1982) delivers the S-curve seen in the data while the model with investment adjustment costs *à la* Christiano et al (2005) does not. (JEL: E32, F41, G15)

Keywords: Capital adjustment costs, Investment adjustment costs, S-curve, International Business Cycles.

1 Introduction

”One of the central questions of international economics concerns the relation between the trade balance and the terms of trade: what features of an economy determine whether an increase in the relative price of imports is associated with improvement or deterioration in the balance of trade?” (Backus, 1993, p. 375)

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This paper attempts to answer Backus's question by focusing on a single class of frictions commonly used in open economy models. We assess the consequences of modeling adjustment costs to capital accumulation for comovements in the trade balance and the terms of trade. We restrict our attention to two types of functions. Investment adjustment costs (IAC), popularized by Christiano, Eichenbaum, and Evans (2005), punish changes in the level of investment. Capital adjustment costs (CAC), described by Hayashi (1982), penalize changes in the capital stock. Most two-country models rely on adjustment costs to capital formation to prevent excessive volatility of investment. This volatility is a consequence of perfect risk sharing. Hence, the benchmark we choose is a two-country, two-good business cycle model with complete markets.

Since the seminal work of Backus, Kehoe, and Kydland (1992), Backus, Kehoe, and Kydland (1995) and Backus, Kehoe, and Kydland (1994) (henceforth BKK), adjustment costs to capital formation have been used extensively in the context of international business cycle models. BKK use a "time-to-build" structure, as in Kydland and Prescott (1982), to dampen the volatility of cross-border investment flows in response to location-specific productivity shocks. However, since the publication of Baxter and Crucini (1995), it has been more common to use Hayashi's (1982) convex capital adjustment costs (e.g. Baxter and Farr, 2005; Yakhin, 2007). Alternative specifications of the capital adjustment cost friction have also been used by Kollmann (1996) and Raffo (2008). Several more recent papers rely on investment adjustment costs to match observed investment volatility. For instance, Enders and Müller (2009) use IAC in an environment with exogenously incomplete markets, while Thoenissen (2011) does so in the complete market setting.

Our main result is that a model featuring capital adjustment costs is consistent with the empirical "S-curved" pattern of cross-correlations between the trade balance and the terms of trade, first observed by Backus, Kehoe, and Kydland (1994), while a model with investment adjustment costs is not. The predictions of the model with capital adjustment costs are robust to the degree of persistence and spillovers of the forcing process, whereas those of the model with investment adjustment costs are not. The two specifications have surprisingly similar predictions for the comovements of quantity aggregates.

To understand the intuition for our result, consider the dynamic responses to a positive productivity shock in the home country. Since each country specializes in the production of a single traded good, a boost to productivity at home results in a relative scarcity of the foreign-produced good. On impact, the price of imports rises relative to the price of exports. As the productivity differences diminish, the terms of trade slowly decline towards their steady-state value. This dynamic response does not depend on the presence or specification of adjustment costs to capital formation. Adjustment costs affect the dynamics of the terms of trade and the trade balance through their effect on the latter.

Under capital adjustment costs, a home productivity shock causes domestic investment to jump on impact. This induces domestic absorption to rise by more than domestic output, causing an immediate deterioration of the trade balance. Net exports then increase as the effect of the productivity shock on investment dissipates. Meanwhile, the terms of trade rise on impact before starting a gradual decline. This implies a negative but increasing correlation between the terms of trade and leads of the trade balance. The cross-correlation function mimics the S-curve pattern seen in the data.

Under investment adjustment costs, investment displays a hump-shaped response to a positive local productivity shock. Domestic absorption inherits from investment its hump-shaped profile. The trade balance in the home country exhibits an inverted hump-shaped profile in response to the productivity shock. As a result, the correlation between the terms of trade and leads of the trade balance declines. This behavior is inconsistent with the S-curve pattern.

While CAC dominates IAC in accounting for the S-curve in our benchmark model, it is worth asking whether this result is specific to our benchmark or a more general feature. We find that our findings are robust to alternative specifications. First, we vary the assumption of complete markets, by restricting financial markets to a single risk-free, one period bond, along the lines of Baxter and Crucini (1995) and Heathcote and Perri (2002). Restricting markets does little to improve the cross-country co-movements of quantity aggregates. As in our benchmark case, with capital adjustment costs we can reproduce the S-curve regularity, while with investment adjustment costs we cannot.

In a second extension, we retain complete markets and develop the environment along the lines of Dmitriev and Roberts (2012) to allow time inseparability of preferences and an arbitrarily small wealth effect on labor supply. In this case, the model resolves the 'international comovement puzzle'. That is, unlike our benchmark and the bond economy, it reproduces the observed positive cross-correlations of investment and employment. Again, capital adjustment costs are consistent with the empirical S-curved pattern, while investment adjustment costs are inconsistent.

Our work contributes to the expanding body of literature that examines the effect of adjustment costs on capital formation. The first branch of this literature estimates the magnitude of adjustment cost using aggregate or industry-wide data. Recent contributions include Hall (2004) and Groth and Khan (2010). The former estimates the magnitude of capital adjustment costs at the industry level, while the latter does the exercise for investment adjustment costs.

A second branch examines the relative performance of CAC and IAC within major classes of DSGE models. For instance, Christiano et al (2005) consider a closed economy New Keynesian model. They show that IAC outperform CAC in accounting for hump-shaped investment responses to monetary shocks. Beaubrun-Diant and Tripier (2005) consider a horse-race between IAC and CAC in a closed economy real business cycle model. They conclude that IAC better accounts for business cycle and asset pricing phenomena than CAC. In contrast, Basu and Thoenissen (2011) find that the observed inverse relationship between the price of investment goods and the investment rate in an international business cycle model driven by TFP shocks is invariant to whether IAC or CAC is used. Our paper contributes to this second branch of the literature. It extends the discussion of the merits of IAC and CAC to a broader consideration of the ability of two-country models driven by productivity shocks to match the data. We conclude that capital adjustment costs do a better job than investment adjustment costs in accounting for international business cycle facts.

Most business cycle models use some form of adjustment costs to moderate the volatility of investment. Yet, our results show that the precise form of this friction influences a models ability to generate the observed features of the data, such as the correlation between the terms of trade and the trade balance. This is important from a modeling perspective for two reasons: First, both capital and investment adjustment costs are commonly used in the open economy macroeconomics literature[1]. Second, the calibration approach employed by most international business cycle models requires that investment volatility predicted by the model be in line with the data. Christiano, Eichenbaum, and Evans (2005) first proposed utilizing investment adjustment costs to reproduce the hump-shaped response of investments to shocks in a closed economy model. The use of investment adjustment costs has since become increasingly popular in both closed and open economy

[1]Recent examples of the models that use CAC include Baxter and Farr (2005), Boileau and Normandin (2008), Devereux and Hnatkovska (2011), Gourio, Siemer, and Verdelhan (2013), Johri, Letendre, and Luo (2011), and Raffo (2008). Models that rely on IAC include Justiniano, Primiceri, and Tambalotti (2010, 2011), Mandelman, Rabanal, Rubio-Ramirez, and Vilan (2011), and Rabanal and Tuesta (2010).

applications. Our results, however, suggest that this specification of adjustment costs should be used with caution in an open economy setting.

The rest of the paper is organized as follows. The next section documents the empirical relationship between the trade balance and the terms of trade. Section 3 describes the model economy. Section 4 reports our quantitative results and discusses how different types of adjustment costs affect the model's ability to reproduce the S-curve. Section 5 presents two extensions. Section 6 offers some concluding remarks.

2 Comovement between the Trade Balance and the Terms of Trade: The S-curve

We start by reviewing the relationship between the trade balance (NX) and the terms of trade (TOT) for recent data. Figure 1 plots cross-correlation functions for $NX(t+k)$ and $TOT(t)$ for k ranging from -8 to 8 quarters. Our samples covers the US, the composite of 15 European countries (EU-15), and the four largest European economies individually. With the exception of the UK, the comovement between the trade balance and the terms of trade exhibits the S-shaped pattern described by BKK. This conclusion holds for two sample lengths.[2]

We would like to emphasize two properties common to the cross-correlation functions for all economies and samples reported:

i.) The correlation between $NX(t+k)$ and $TOT(t)$ is negative for $k = 0$, and for several first positive lags k;

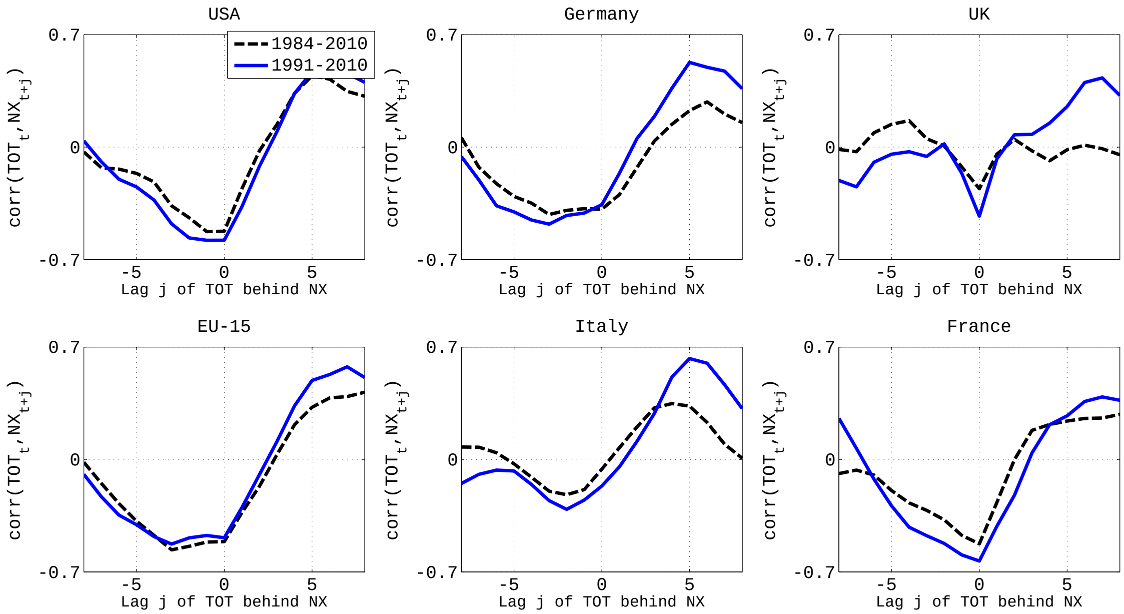
ii.) The correlation between $NX(t+k)$ and $TOT(t)$ is increasing for the few first positive lags k.

In the rest of the paper, we will rationalize these properties by referring to the transmission mechanism of technology shocks discussed by BKK. We will show that while capital adjustment costs are consistent with the S-curve, investment adjustment costs are not.

We are not the first to report the S-shaped cross-correlations between the trade balance and the terms of trade. In addition to BKK, who coined the term "S-curve", the related literature includes

[2] BKK showed that the pattern of cross-correlations between TOT and NX might depend on the sampling period. Therefore, we consider a sample covering 1984:1-2010:2 and its subsample covering 1991:1-2010:2. Our subsample starts at the end of BKK's sample.

Figure 1: The S-curve: Cross-Correlation Function of the Trade Balance and the Terms of Trade



Note: The figure depicts the cross-correlation function between leads and lags of net exports and the terms of trade for five OECD countries and the EU-15. Data come from the OECD Quarterly National Accounts. NX denotes the ratio of Net Exports to GDP measured in current prices. TOT is the logarithm of the ratio of the import deflator to the export deflator. All series are HP-filtered with a smoothing parameter of 1600. European data (EU-15) cover the following 15 countries: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal, Spain, Sweden, and the United Kingdom. The data are at quarterly frequency, seasonally adjusted and aggregated at the source.

Senhadji (1998) who documents an S-shaped relation between leads and lags of the two variables for emerging economies. More recently, Bahmani-Oskooee and Ratha (2007) describe this regularity between US and its trading partners. Enders and Müller (2009) document both conditional and unconditional S-curves for recent US data.

3 The Model Economy

The economic environment we consider consists of two countries. The same parameters describe technology and preferences in both countries. Each country $j = 1, 2$ is populated by a continuum of identical infinitely lived individuals. Labor is immobile across countries. In each period t, the world economy experiences an event s_t drawn from the countable set of events, S. Let $s^t = (s_0, s_1, ..., s_t) \in S^t$ be the history of events from time 0 to time t. The probability at time 0 of any given history s^t is denoted by $\pi\left(s^t\right)$.

3.1 Capital accumulation with adjustment costs

Households in country $j \in \{1,2\}$ own the domestic capital stock, $k_j\left(s^t\right)$, and rent it to domestic intermediate good firms. Installed capital is immobile across countries.

3.1.1 Investment adjustment costs

When the model features the investment adjustment costs of Christiano, Eichenbaum, and Evans (2005), the stock of capital evolves according to the following law of motion:

$$k_j\left(s^t\right) = (1-\delta)k_j\left(s^{t-1}\right) + \psi\left(i_j\left(s^t\right), i_j\left(s^{t-1}\right)\right), \text{ for } j \in \{1,2\}, \tag{1}$$

where δ is the depreciation rate, $i_j\left(s^t\right)$ denotes gross investment, and

$$\psi\left(i_j\left(s^t\right), i_j\left(s^{t-1}\right)\right) = \left[1 - \Delta\left(\frac{i_j\left(s^t\right)}{i_j\left(s^{t-1}\right)}\right)\right] i_j\left(s^t\right).$$

Imposing restrictions $\Delta(1) = \Delta'(1) = 0$ ensures that the steady state of the model is independent of the adjustment cost. We adopt the following functional form from Schmitt-Grohe and Uribe (2004):

$$\Delta(x) = \frac{\chi}{2}(x-1)^2,$$

where $\chi = \Delta''(\cdot) > 0$ controls the penalty for changing the level of investment. This formulation implies that it is costly to change the level of investment. Moreover, the cost is increasing in the magnitude of the change.

3.1.2 Capital adjustment costs

When capital accumulation is subject to the convex adjustment costs described in Hayashi (1982), the law of motion for the capital stock reads as

$$k_j\left(s^t\right) = (1-\delta)k_j\left(s^{t-1}\right) + \phi\left(\frac{i_j\left(s^t\right)}{k_j\left(s^{t-1}\right)}\right) k_j\left(s^{t-1}\right), \text{ for } j \in \{1,2\}. \tag{2}$$

The adjustment cost function ϕ satisfies $\phi(\cdot) > 0$, $\phi'(\cdot) > 0$, and $\phi''(\cdot) < 0$. Since we do not rely on log-linearization methods for solving the model, we must specify the functional form for capital adjustment costs explicitly. We adopt the following formulation used by Boldrin, Christiano, and

Fisher (2001)

$$\phi\left(x\right)=\frac{\kappa_{1}}{1-1/\xi}\left(x\right)^{1-1/\xi}+\kappa_{2},$$

where $\kappa_1=\delta^{1/\xi}$, $\kappa_2=\delta/(1-\xi)$, and ξ is the elasticity of investment with respect to Tobin's q. The restrictions $\phi\left(\delta\right)=\delta$ and $\phi'\left(\delta\right)=1$ imposed on the constants κ_1 and κ_2 ensure that incorporation of the adjustment cost does not affect the deterministic steady state of the model. This formulation implies that it is costly to change the level of capital. Once again, the cost is increasing in the magnitude of the change.

3.2 Producers

The are two types of firms in this environment: intermediate good producers and final good producers. Intermediate good firms in each country specialize in the production of a single country-specific good. Country 1 produces good a, whereas Country 2 produces good b. Households supply labor to the intermediate firms at the wage $w_j\left(s^t\right)$, and capital at the rental price $r_j\left(s^t\right)$. The firms have access to a constant returns-to-scale technology. Production is subject to a country-specific exogenous random shock $z_j\left(s^t\right)$ to total factor productivity (TFP). Output in country j after history s^t is given by

$$y_j\left(s^t\right)=z_j\left(s^t\right)f(k_j\left(s^{t-1}\right),n_j\left(s^t\right)), \tag{3}$$

where f is a Cobb-Douglas production function given by $f(k,n)=k^{\alpha}n^{1-\alpha}$. The TFP shocks follow a stationary vector autoregressive process in logs:

$$\begin{bmatrix}\log(z_1\left(s^t\right))\\ \log(z_2\left(s^t\right))\end{bmatrix}=\begin{bmatrix}\rho_{11} & \rho_{12}\\ \rho_{12} & \rho_{11}\end{bmatrix}\begin{bmatrix}\log(z_1\left(s^{t-1}\right))\\ \log(z_2\left(s^{t-1}\right))\end{bmatrix}+\begin{bmatrix}\varepsilon_1\left(s^t\right)\\ \varepsilon_2\left(s^t\right)\end{bmatrix}.$$

Diagonal elements of the transition matrix, ρ_{11}, determine the degree of persistence in productivity within each country. Off-diagonal elements, ρ_{12}, determine the speed with which productivity innovations spill over national borders. The innovations to the productivity process are serially uncorrelated bivariate normal random variables with zero mean and the contemporaneous covariance matrix

$$E\left[\varepsilon_t\varepsilon_t'\right]=\sigma_{\varepsilon}^2\cdot\begin{bmatrix}1 & \rho_{\varepsilon}\\ \rho_{\varepsilon} & 1\end{bmatrix}.$$

Final good firms in country 1 use as inputs the quantities $a_1\left(s^t\right)$ and $b_1\left(s^t\right)$ of the two intermediate goods. They produce an amount $G\left(a_1\left(s^t\right), b_1\left(s^t\right)\right)$ of the final good, using a constant returns-to-scale technology

$$G\left(a, b\right) = \left(\omega a^{1-\eta} + \left(1-\omega\right) b^{1-\eta}\right)^{\frac{1}{1-\eta}},$$

where $1/\eta$ is the elasticity of substitution between intermediate inputs. This is the standard Armington aggregator function used by BKK and Heathcote and Perri (2002), among others, in the context of international business cycle models. The constant $\omega \in (1/2, 1)$ reflects the extent of home bias. In the same way, the firms in country 2 produce an amount $G\left(b_2\left(s^t\right), a_2\left(s^t\right)\right)$ of the final good, using $a_2\left(s^t\right)$ and $b_2\left(s^t\right)$ of the intermediate inputs.

3.3 Consumers

Consumers have their preferences defined over stochastic sequences of consumption, and leisure

$$U = \sum_{t=0}^{\infty} \beta^t \sum_{s^t \in S^t} \pi\left(s^t\right) u(c_j\left(s^t\right), n_j\left(s^t\right)), \tag{4}$$

where $\beta \in (0, 1)$ is the discount factor. Let $c_j\left(s^t\right)$ denote household consumption in country j after realization of history s^t, and let $n_j\left(s^t\right) \in [0, 1]$ denote individual labor supply. Time endowment per period is normalized to one. The instantaneous utility function takes the following form:

$$u\left(c, n\right) = \frac{\left[c^{\gamma}(1-n)^{1-\gamma}\right]^{1-\sigma} - 1}{1-\sigma},$$

where σ is the curvature parameter, and γ determines the relative importance of leisure and consumption.

Agents have access to a complete set of state-contingent claims traded internationally. The claims are denominated in terms of the intermediate good a. Let $Q\left(s^t, s_{t+1}\right)$ be the price of a claim sold after realization of s^t, that delivers a unit of good a at time $t+1$ provided that the state s_{t+1} is realized. Let $B_2\left(s^t, s_{t+1}\right)$ be the quantity of such claims held by the residents of country 2 after history s^t. The consumers in country 2 then face the following budget constraint:

$$\begin{aligned} & c_2\left(s^t\right) + i_2\left(s^t\right) + q_2^a\left(s^t\right) \sum_{s_{t+1} \in S} Q\left(s^t, s_{t+1}\right) B_2\left(s^t, s_{t+1}\right) \\ = \; & q_2^b\left(s^t\right) \left[r_2\left(s^t\right) k_2\left(s^{t-1}\right) + w_2\left(s^t\right) n_2\left(s^t\right)\right] + q_2^a\left(s^t\right) B_2\left(s^{t-1}, s_t\right), \end{aligned} \tag{5}$$

where $q_2^a\left(s^t\right)$ and $q_2^b\left(s^t\right)$ denote the prices of goods a and b respectively, in terms of the final good of country 2. The agents in country 1 face a similar constraint.

3.4 Equilibrium

The equilibrium consists of the state-contingent sequences of factor prices $\left\{\left\{r_j\left(s^t\right), w_j\left(s^t\right)\right\}_{j\in\{1,2\}}\right\}_{t=0,s^t\in S^t}^{\infty}$, intermediate good prices $\left\{\left\{q_j^a\left(s^t\right), q_j^b\left(s^t\right)\right\}_{j\in\{1,2\}}\right\}_{t=0,s^t\in S^t}^{\infty}$, bond prices $\left\{\left\{Q\left(s_{t+1}, s^t\right)\right\}_{s_{t+1}\in S}\right\}_{t=0,s^t\in S^t}^{\infty}$ and allocations $\left\{\left\{c_j\left(s^t\right), i_j\left(s^t\right), n_j\left(s^t\right), k_j\left(s^t\right), a_j\left(s^t\right), b_j\left(s^t\right), \left\{B_j\left(s_{t+1}, s^t\right)\right\}_{s_{t+1}\in S}\right\}_{j\in\{1,2\}}\right\}_{t=0,s^t\in S^t}^{\infty}$ that satisfy the following conditions. Given the prices:

i) Consumers in country j choose state contingent sequences $\{c_j(s^t)\}_{t=0}^{\infty}$, $\{n_j(s^t)\}_{t=0}^{\infty}$, $\{i_j(s^t)\}_{t=0}^{\infty}$ and bond holdings $\{B_j(s_{t+1}, s^t)\}_{s_{t+1}\in S}$ for all $s^t \in S^t$, to maximize expected utility (4) subject to the budget constraint (5), the corresponding law of motion for capital (2) or (1), and the initial conditions.

ii) Intermediate firms in country j choose $n_j(s^t)$ and $k_j(s^{t-1})$ to maximize profits

$$y_j(s^t) - r_j(s^t)k_j(s^{t-1}) - w_j(s^t)n_j(s^t),$$

subject to the technological constraint (3), and the non-negativity constraints $n_j(s^t) \geq 0$ and $k_j(s^{t-1}) \geq 0$.

iii) The final good producers in country j choose $a_j\left(s^t\right)$ and $b_j\left(s^t\right)$ to maximize profits $\Pi_j\left(s^t\right)$ given by

$$\Pi_j\left(s^t\right) = \begin{cases} G\left(a_j\left(s^t\right), b_j\left(s^t\right)\right) - q_j^a(s^t)a_j(s^t) - q_j^b(s^t)b_j(s^t), & \text{for} \quad j = 1; \\ G\left(b_j\left(s^t\right), a_j\left(s^t\right)\right) - q_j^a(s^t)a_j(s^t) - q_j^b(s^t)b_j(s^t), & \text{for} \quad j = 2. \end{cases}$$

The prices are such that for all $t \geq 0$ and for all $s^t \in S^t$:

iv) Intermediate good markets clear

$$\begin{aligned} a_1\left(s^t\right) + a_2\left(s^t\right) &= y_1(s^t), \\ b_1\left(s^t\right) + b_2\left(s^t\right) &= y_2(s^t). \end{aligned}$$

v) Final good markets clear

$$c_1\left(s^t\right) + i_1\left(s^t\right) = G\left(a_1\left(s^t\right), b_1\left(s^t\right)\right),$$
$$c_2\left(s^t\right) + i_2\left(s^t\right) = G\left(b_2\left(s^t\right), a_2\left(s^t\right)\right).$$

vi) Asset markets clear

$$B_1(s_{t+1}, s^t) + B_2(s_{t+1}, s^t) = 0, \text{ for all } s_{t+1} \in S. \tag{6}$$

3.5 Calibration and Solution

3.5.1 Parameter Values

We numerically solve the models using the parameter values reported in Table 1. The first group of parameters takes values common to the international business cycle literature. We adopt from BKK the values for the capital-income share α, the utility curvature σ, and the parameters governing the stochastic process for productivity. Similar to BKK, we set the elasticity of substitution between intermediate goods $1/\eta$ to 1.5. In the variations, we consider the sensitivity of our results to the parameterization of the forcing process and the elasticity of substitution $1/\eta$.

Table 1: Baseline Parameter Values

Preferences:	Discount factor	β	0.989
	Consumption share	γ	0.361
	Utility curvature	σ	2
Technology:	Capital income share	α	0.36
	Depreciation rate	δ	0.025
	Import share	im	0.15
	Elasticity of substitution between intermediate goods	$1/\eta$	1.5
Productivity:	Persistence of productivity shocks	ρ_{11}	0.906
	Spillover parameter	ρ_{12}	0.088
	St. dev. of innovations to productivity	σ_ε	0.00852
	Correlation of innovations to productivity	ρ_ε	0.258

Note: The time period is a quarter of a year. The parameters ξ and χ that control the degree of adjustment costs are set to match the relative standard deviation of investment in the data.

We consider a symmetric steady-state in which $y_1 = y_2$ and $b_1 = a_2$. The steady-state import share $im = b_1/y_1$ is set to 0.15 as in BKK and Heathcote and Perri (2002). The parameter ω, that governs the degree of home-bias follows from the expression for the terms of trade in the steady state

$$TOT = \frac{1-\omega}{\omega}\left(\frac{1-im}{im}\right)^{\eta}. \tag{7}$$

Because $TOT = 1$, equation (7) pins down the value of ω for a given value of the trade elasticity $1/\eta$.

Other parameters are calibrated to match long-run averages in the US data as described in Cooley (1997). One period of time corresponds to one quarter. The quarterly depreciation rate δ ensures that the steady-state investment to GDP ratio is 0.25 and the capital to GDP ratio is 10. Once δ is set, the discount factor β follows directly from first-order conditions for the consumer's optimization problem in the steady state. Given the values of α, δ and the steady-state capital-output ratio k/qy, we compute the discount factor as $\beta = (\alpha\,(qy/k)+1-\delta)^{-1}$. The weight of leisure in the instantaneous utility function, $1-\gamma$, ensures that the steady-state level of hours worked n remains at 1/3.

3.5.2 Solution Method

We solve the model numerically using an Euler equation-based method that does not require linearization of the optimality conditions[3]. The algorithm replaces conditional expectations in the first order conditions by smooth parametric functions of the current state variables and iterates on the parameter values until the rational expectations equilibrium is achieved We solve the model numerically using a variant of the ergodic set methods described by Maliar, Maliar, and Judd (2011). The algorithm used is classified by Judd, Maliar, and Maliar (2009) as belonging to the stochastic simulation class of methods. This approach has three key advantages. First, it allows us to avoid computing solutions in areas of the state space that are never visited in equilibrium. Second, it enables us to avoid the approximation errors associated with linearization of the optimality conditions. Third, as Kollmann, Maliar, Malin, and Pichler (2011) demonstrate, it can handle a large number of state variables with lower computational cost than competing methods.[4]

[3]Supplementary materials available at `www.admitriev.net` show how to solve the model with more conventional methods e.g. those imbedded in DYNARE.

[4]In terms of accuracy, stochastic simulation approaches occupy a middle ground between perturbation methods and projection methods. Further discussion of our approach and comparisons with other solution techniques can be found in a recent special issue of the *Journal of Economic Dynamics and Control* (see den Haan, Judd, and Juillard, 2011).

The algorithm parameterizes decision rules derived from the first order conditions using a polynomial approximation function of the current state variables. It then uses a fixed point iteration method to compute fixed-point values of the polynomial coefficients, evaluating conditional expectations using a combination of Monte-Carlo integration and regression.

4 The Result

This section compares comovement patterns between the terms of trade and the trade balance predicted by the models with the two types of adjustment costs. We measure the terms of trade in country 1 as

$$TOT\left(s^t\right) = q_1^b(s^t)/q_1^a(s^t).$$

We define the trade balance as the net exports for country 1 as a fraction of GDP for country 1, both measured in current prices

$$NX\left(s^t\right) = \frac{q_1^a(s^t)a_2(s^t) - q_1^b(s^t)b_1(s^t)}{q_1^a(s^t)y_1(s^t)}.$$

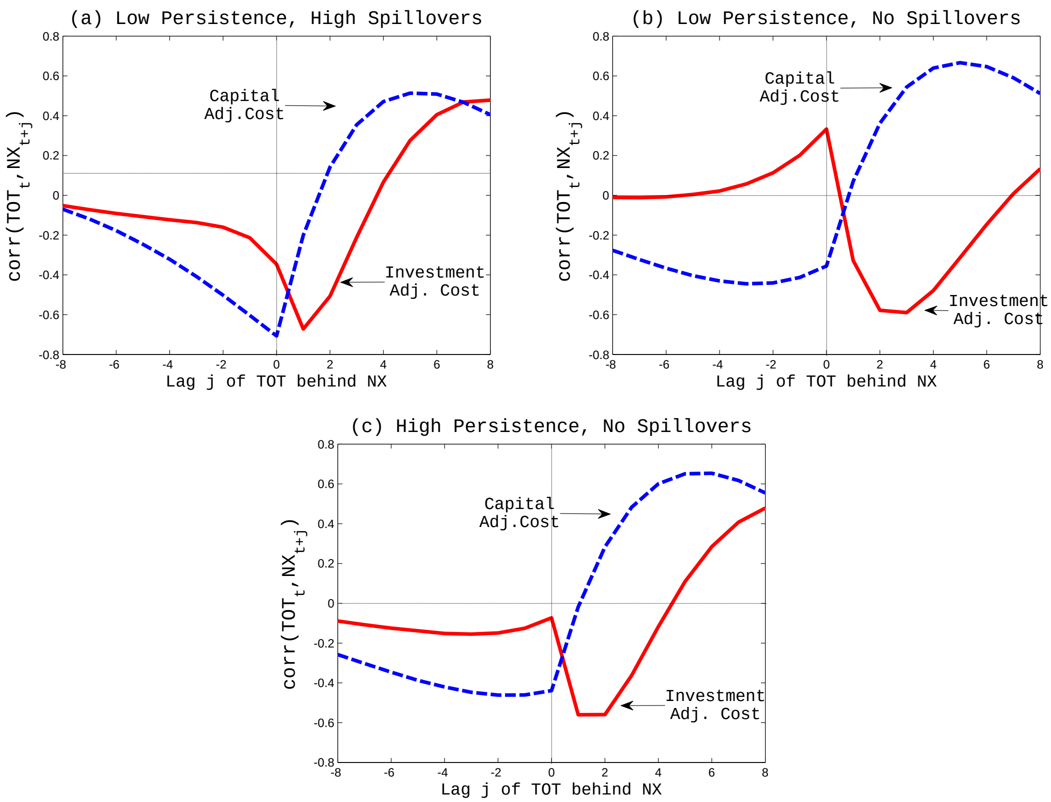
4.1 Does the Introduction of Adjustment Costs Preserve the S-curve?

Our simulation results, reported in Figure 2 and Table 2, can be summarized as follows. First, the introduction of investment adjustment costs violates the S-shaped pattern of cross-correlations between $NX(t+k)$ and $TOT(t)$. Depending on the specification of the forcing process, the contemporaneous correlation of the terms of trade and the trade balance can be positive or negative. This correlation is negative in the data. The graph of the predicted cross-correlation function is U-shaped for k ranging between 0 and 4. Yet, it is strictly increasing in the data. Second, the model with capital adjustment costs preserves the S-shaped cross-correlation pattern between $NX(t+k)$ and $TOT(t)$ observed in Figure 1. Third, models with either type of adjustment cost deliver remarkably similar predictions for most business-cycle statistics. Finally, the predictions of the model with IAC are sensitive to the parameterization of the forcing process, whereas the predictions of the model with CAC are not.

4.2 Persistence, Spillovers and the S-curve

Predictions of the international business cycle models are known to be sensitive to the specification of the stochastic process for TFP (Heathcote and Perri, 2002). We consider three variations in the parameterization of the forcing process. They differ in the degree of persistence and the extent to which the innovations spill over national borders.

Figure 2: The S-curve? Predictions of the Models with Capital Adjustment Costs and Investment Adjustment Costs



Note: The figure depicts cross-correlation functions between the trade balance and the terms of trade implied by the models with adjustment costs. Panels (a)-(c) differ in the specification of the forcing process.

Our benchmark model uses BKK's estimate of the TFP process, which we consider least favorable to our argument. The reason is simple. Compared to other estimates, the BKK process features relatively low persistence of shocks (0.906) and relatively high spillovers (8.8% per quarter). Increasing persistence or reducing spillovers makes productivity differentials between countries more long-lasting. Therefore, for a given degree of adjustment cost, investment becomes more volatile. To keep the relative volatility of investment to output at the target level, we have to increase the cost of adjustment. The latter makes the differences between CAC and IAC even more pronounced. Figure 2 illustrates this point by comparing the S-curves implied by models with different parameterizations of spillovers and persistence. Panel (b) compares the models' predictions when cross-country spillovers are absent. Panel (c) reports the S-curves implied by the models with near-unit-root persistence. In both cases, the predictions of the IAC model for the S-curve are inferior compared with those of the CAC model.

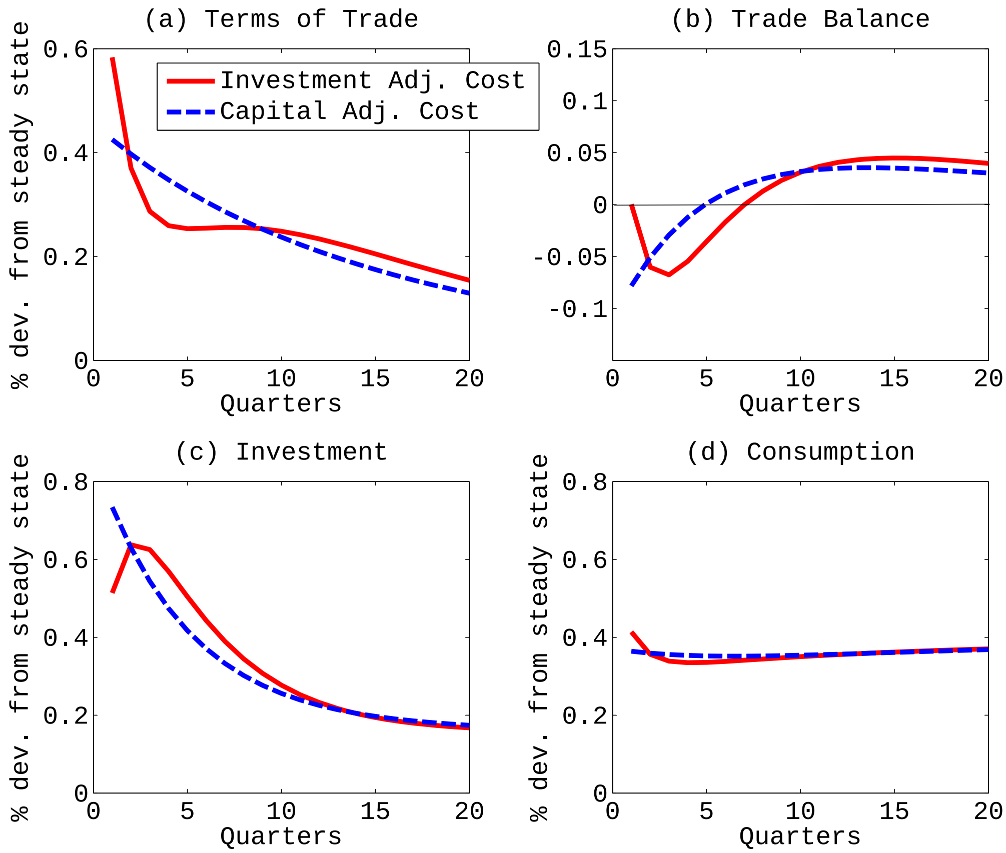
4.3 Intuition

To understand the intuition for our result, consider responses to a one-standard-deviation positive productivity shock in Country 1 depicted in Figure 3 and Figure 4. An increase in domestic productivity leads to a relative abundance of the home-produced good. The price of imported goods rises relative to the price of exports, which leads to a sudden increase in the terms of trade. As productivity differences disappear, the terms of trade slowly return to their steady state level. This response is common to the models with both types of adjustment cost (Figure 3a). However, the responses of the trade balance are very different.

Consider the reactions of the aggregate quantities in the aftermath of the shock. On impact, consumption jumps and begins its very slow decline due to the high persistence of shocks. Domestic wages increase and so does domestic output, since the substitution effect dominates the wealth effect. Both features are shared by the two models. What differs most is the behavior of investment. These differences translate into qualitatively different responses of NX to the productivity shocks.

Under capital adjustment costs, changes in the capital stock are progressively penalized. The optimal response of domestic investment is a jump, followed by a gradual decline as productivity differences vanish (Figure 3c). As a result, domestic absorption jumps by more that domestic output (Figure 4b), causing a trade deficit (Figure 3b). Eventually the wedge between the two disappears as investment declines (Figure 4b). These dynamics are reflected in the evolution of the trade balance in Figure 3b. In the model with capital adjustment costs, the trade balance deteriorates on impact. It immediately starts to improve as domestic absorption declines, led by investment. This explains the negative but increasing correlation between $TOT(t)$ and $NX(t+k)$ for $k=0$ and for low positive values of k.

Figure 3: Responses to a TFP Shock in Country 1: TOT, NX, Investment and Consumption

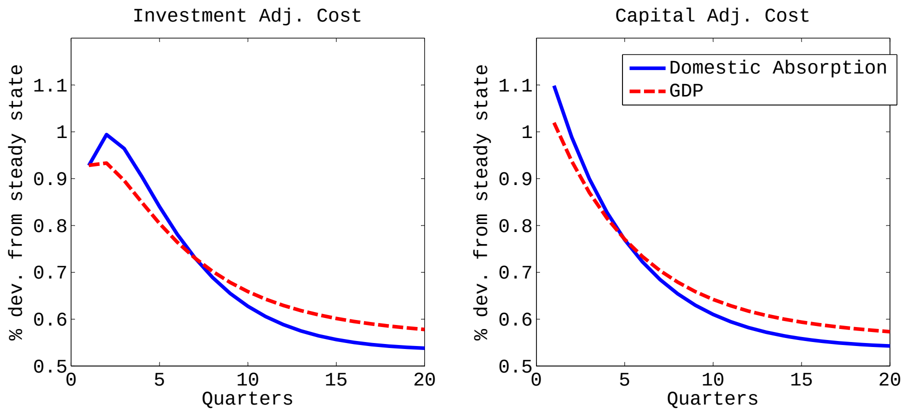


Note: Panels (a) and (b) depict percentage deviations of variables from their own steady state values. Panels (c) and (d) show deviations of consumption and investment from their steady states as a percentage of output's steady-state value.

Under investment adjustment costs, changes in the level of investment are progressively penalized. The optimal response of domestic investment to a positive shock is hump-shaped (Figure 3c). Domestic absorption inherits from investment its hump-shaped profile. Output's response mimics that of productivity: a jump is followed by a gradual decline. Therefore, the response of the trade balance, measured as the wedge between output and absorption, is virtually zero on impact. In absolute terms, its volume reaches a maximum in a few quarters, just after investment peaks. Then, it begins a slow decline towards the steady-state level. As depicted in Figure 3b, the response of net exports to a positive shock exhibits an inverted hump-shaped profile. However, the response of the terms of trade is a jump followed by a steady decline. As a result, the cross-correlation function

$TOT(t)$ and $NX(t+k)$ declines after $k = 0$ and reaches its minimum only after a few quarters. These dynamics are inconsistent with the S-curve reported in Figure 1.

Figure 4: Responses to a TFP Shock in Country 1: Domestic Absorption and GDP



Note: Both panels depict deviations of the variables from their respective steady states as a percentage of output's steady-state value.

The hump-shaped response of investment to a shock is ultimately responsible for the counterfactual predictions of a comovement between NX and TOT. Unfortunately, it is precisely this feature that has made IAC attractive for several classes of models.

4.4 Extensions

We consider two extensions to our benchmark model. Our first extension departs from the complete market assumption. The second extension departs from the assumption of the time-separability of preferences. Our main result holds in both cases. Moreover, the second extension performs at least as well as incomplete market models in accounting for business cycle properties of aggregate quantities.

4.4.1 An Incomplete Market Economy

As an extension to our benchmark model we consider an incomplete market environment in the spirit of Baxter and Crucini (1995) and Heathcote and Perri (2002). We restrict the span of internationally traded securities to a one-period riskless bond. The setup follows Heathcote and Perri (2002).

Agents have access to one-period uncontingent claims traded internationally. The riskless bonds are denominated in terms of the intermediate good a. Let $Q\left(s^t\right)$ be the price of a bond sold after realization of s^t, that delivers a unit of good a at time $t+1$ irrespective of realization of state s_{t+1}. Denoting by $B_1\left(s^t\right)$ the quantity of such claims held by the residents of country 1 after history s^t, we can write their budget constraint as follows:

$$\begin{aligned} & c_1\left(s^t\right)+i_1\left(s^t\right)+q_1^a\left(s^t\right)Q\left(s^t\right)B_1\left(s^t\right) \\ = \; & q_1^a\left(s^t\right)\left[r_1\left(s^t\right)k_1\left(s^{t-1}\right)+w_1\left(s^t\right)n_1\left(s^t\right)\right]+q_1^a\left(s^t\right)\left[B_1\left(s^{t-1}\right)-\Phi\left(B_1\left(s^t\right)\right)\right], \end{aligned} \tag{8}$$

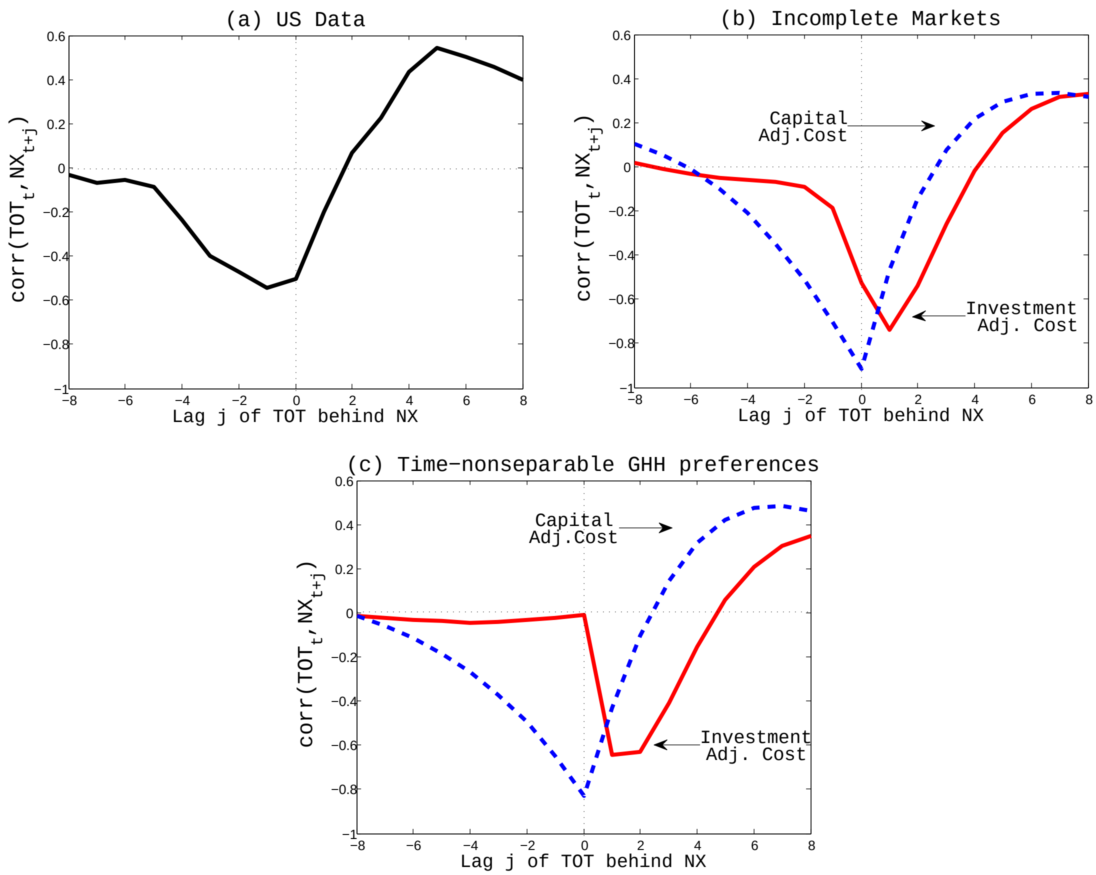
To induce stationarity, we impose the arbitrarily small cost of holding bonds measured in units of good a. Following Heathcote and Perri (2002), we assume the quadratic adjustment cost of bond holding: $\Phi\left(B_1\left(s^t\right)\right)=\frac{\phi}{2}\left(B_1\left(s^t\right)\right)^2$. The equilibrium is defined as before (section 3.4), except that the asset market clearing now requires that $B_1\left(s^t\right)+B_2\left(s^t\right)=0$

Our findings can be summarizes as follows. The main result holds in the incomplete market environment. The model featuring capital adjustment costs is consistent with the S-curve pattern of cross-correlations between net exports and the terms of trade observed in the data, while the model with investment adjustment costs is not. The reasons that our main result holds appear to be the same as in the benchmark model. As have already been noted by Heathcote and Perri (2002) restricting international lending to a one-period riskless bond helps little to account for the cross-country comovement of quantity aggregates.

4.4.2 A Model with Time-nonseparable Preferences that Allow Arbitrarily Small Wealth Effects on Labor Supply

We extend the complete markets setting by introducing habit formation in consumption and preference structure popularized by Greenwood, Hercowitz, and Huffman (1988) (henceforth GHH). As shown by Dmitriev and Roberts (2012), relaxing the assumption of time separable preferences and reducing the size of the wealth effect of labor supply allows a two-country, one-good model to resolve the 'international co-movement puzzle'. This puzzle refers to the fact that the cross-correlations between investment and employment are positive in the data, whereas business cycle models tend to predict that they are negative.

Figure 5: The S-curve? Extensions to the Baseline Model



Note: The figure depicts cross-correlation functions between the trade balance and the terms of trade implied by the models with adjustment costs. Panel (a) corresponds to the US data; panel (b) predictions of the model with incomplete markets; panel (c) predictions of the model with time-nonseparable GHH preferences.

Dmitriev and Roberts (2012) only considered a one-good environment. To consider the S-curve relationship, and allow comparison with the benchmark and incomplete markets models, we introduce the same class of preferences to a two-country, two-good environment. We assume that the households in country j maximize

$$\sum_{t=0}^{\infty} \beta^t \sum_{s^t \in S^t} \pi\left(s^t\right) u(\bar{c}_j\left(s^t\right), n_j\left(s^t\right)), \tag{9}$$

where $\bar{c}_j\left(s^t\right)$ denotes habit-adjusted consumption. It is defined as a quasi-difference between the agent's current consumption and the previous period's consumption $\bar{c}_j\left(s^t\right) = c_j\left(s^t\right) - Bc_j(s^{t-1})$ as in Constantinides (1990). The instantaneous utility function defined over habit-adjusted consumption

$\bar{c}$ and hours worked n is adopted from Greenwood, Hercowitz, and Huffman (1988)

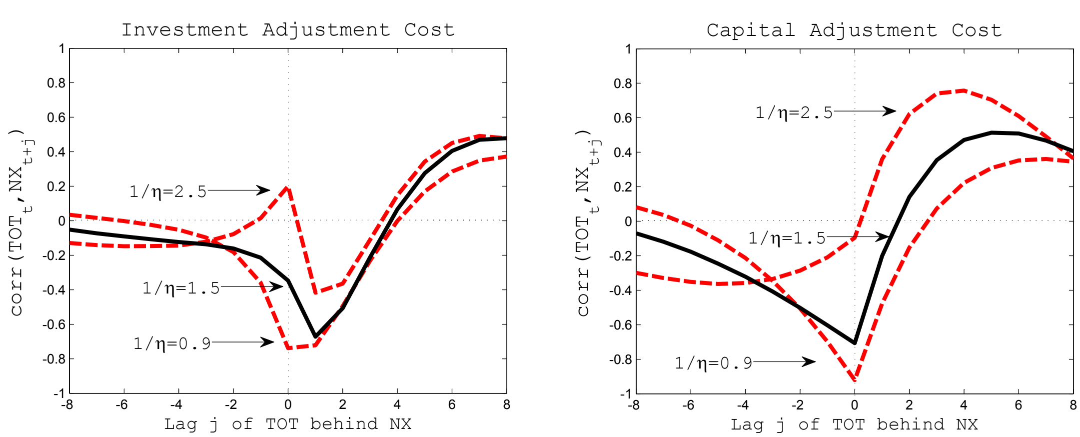
$$u(\bar{c}, n) = \frac{1}{1-\sigma}\left(\bar{c} - \chi\frac{n^{1+\nu}}{1+\nu}\right)^{1-\sigma}.$$

Our findings are as follows. First, the model accounts for positive cross-country comovement of investment and hours worked ("international comovement puzzle") as in a one-good model of Dmitriev and Roberts (2012). In addition, it accounts for countercyclical net export and predicts realistic cross-country correlation of hours worked. It performs at least as well as incomplete market models driven by productivity shocks in accounting for second moments of quantity aggregates. Second, our main result still holds. Once again, the model featuring capital adjustment costs is consistent with the S-curve pattern of cross-correlations between net exports and the terms of trade observed in the data, while the model with investment adjustment costs is not. Finally, the reason seems to be the same as in the models with standard preferences.

4.5 Sensitivity Experiments

Since the seminal work of BKK it has been known that quantitative predictions of two-country two-good business cycle models depend on the elasticity of substitution between home and foreign produced goods. In this section, we explore how variation in the elasticity parameter affects our conclusion for the relationship between the trade balance and the terms of trade.

Figure 6: Elasticity of Substitution between Intermediate Goods and the S-curve



Note: The figure shows the sensitivity of the cross-correlation function between the trade balance and the terms of trade to variations in the elasticity of substitution between intermediate goods.

The results of the sensitivity experiments are reported in Figure 5. They can be summarized as follows. First, predictions of the model with IAC are relatively more sensitive to the elasticity of substitution $1/\eta$. Second, for a reasonably high value of the trade elasticity, $1/\eta = 2.5$, a model with IAC delivers positive comovement of NX and TOT. The opposite is seen in the data (see Figure 1), and is predicted by the model with CAC. Finally, both models perform better when the trade elasticity is low. The reason is fairly intuitive. Higher substitutability between domestic and foreign tradable goods makes our model resemble a single good economy. What BKK (1995, p. 340) call the "make hay where the sun shines" motive for shifting investment to the most productive location becomes stronger. Hence, a higher degree of adjustment cost is required to ensure that investment volatility remains at the observed level. The latter distorts the S-curve.

5 Concluding Remarks

The S-shaped relation between leads and lags of the trade balance and the terms of trade is a well-established feature of the international data. It is desirable for international business cycle models to reproduce this regularity. Most two-country models rely on adjustment costs to capital formation to reproduce the observed volatility of investment. The choice of adjustment costs has consequences for a model's ability to deliver the S-curve. We show that a complete markets model with capital adjustment costs is consistent with the S-curve, while a model with investment adjustment costs is not. The specification of adjustment costs has remarkably little effect on the ability of the model to reproduce international comovements.

In extensions, we (i) restrict financial markets to a single uncontingent bond and (ii) retain complete markets, but adapt the model along the lines of Dmitriev and Roberts (2012) to allow time non-separability of preferences and an arbitrarily small wealth effect on labor supply. The second extension results in improved predictions for business cycle statistics. Yet in accounting for the S-curve, capital adjustment costs still dominate investment adjustment costs in both environments.

We emphasize that the model featuring investment adjustment costs is inconsistent with an S-curve relationship because the dynamic response of investment to shocks is hump-shaped. It is this very feature that has led to the widespread adoption of investment adjustment costs in several classes of models in recent years. We have highlighted that this feature comes at a cost in two-country business cycle models.

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A Technical Appendix

This appendix presents the system of equations that characterize equilibrium for the benchmark model with complete markets and an extension with incomplete markets.

A.1 Optimality conditions

This section first describes the optimality conditions shared by the baseline models in the complete market environment. Then how the introduction of adjustment costs alters the Euler equations and the equations of motions for capital is discussed. Finally, how consideration of an incomplete market setting changes the common optimality conditions is shown. In this appendix, $c_j(s^t)$ and $k_j(s^{t-1})$ are denoted as c_{jt} and k_{jt-1} respectively. The same notational convention applies to the rest of the variables.

A.1.1 Common Optimality Conditions: Complete Markets

Regardless of whether a model features adjustment costs, the complete market environment is characterized by the following set of optimality conditions:

(i) Marginal utilities of consumption

$$\Lambda_{jt} = \frac{\gamma}{c_{jt}} \left[c_{jt}^{\gamma} (1 - n_{jt})^{1-\gamma} \right]^{1-\sigma}, \text{ for } j = 1, 2, \tag{10}$$

(ii) Labor-supply equations

$$\frac{\gamma}{(1-\gamma)}\frac{c_{1t}}{(1-n_{1t})}=q_{1t}^{a}w_{1t}, \tag{11}$$

$$\frac{\gamma}{(1-\gamma)}\frac{c_{2t}}{(1-n_{2t})}=q_{2t}^{b}w_{2t}, \tag{12}$$

(iii) Demand for intermediate goods

$$a_{1t}=\omega^{1/\eta}\,(q_{1t}^{a})^{-1/\eta}\,Y_{1t}, \tag{13}$$

$$a_{2t}=(1-\omega)^{1/\eta}\,(q_{1t}^{a}/RER_t)^{-1/\eta}\,Y_{2t}, \tag{14}$$

$$b_{2t}=\omega^{1/\eta}\left(q_{2t}^{b}\right)^{-1/\eta}Y_{2t}, \tag{15}$$

$$b_{1t}=(1-\omega)^{1/\eta}\left(RER_t\cdot q_{2t}^{b}\right)^{-1/\eta}Y_{1t}, \tag{16}$$

(iv) Production functions of the final goods

$$Y_{1t}=\left(\omega a_{1t}^{1-\eta}+(1-\omega)\,b_{1t}^{1-\eta}\right)^{\frac{1}{1-\eta}}, \tag{17}$$

$$Y_{2t}=\left((1-\omega)\,a_{2t}^{1-\eta}+\omega b_{2t}^{1-\eta}\right)^{\frac{1}{1-\eta}}, \tag{18}$$

(v) Market clearing for the final goods

$$c_{jt}+i_{jt}=Y_{jt},\ \text{for } j=1,2, \tag{19}$$

(vi) Market clearing for intermediate goods

$$a_{1t}+a_{2t}=z_{1t}k_{1t-1}^{\alpha}n_{1t}^{1-\alpha}, \tag{20}$$

$$b_{1t}+b_{2t}=z_{2t}k_{2t-1}^{\alpha}n_{2t}^{1-\alpha}, \tag{21}$$

(vii) Factor prices

$$r_{jt}=\alpha z_{jt}k_{jt-1}^{\alpha-1}n_{jt}^{1-\alpha},\ \text{for } j=1,2, \tag{22}$$

$$w_{jt}=(1-\alpha)\,z_{jt}k_{1t-1}^{\alpha}n_{jt}^{-\alpha},\ \text{for } j=1,2, \tag{23}$$

(viii) Risk-sharing condition

$$\Lambda_{1t}RER_t=\Lambda_{2t}. \tag{24}$$

A.1.2 Model without Adjustment Costs

In addition to conditions (i)–(viii), the optimality conditions of the model with no adjustment costs include

(ix) Euler equations

$$m_{2t} = \beta E_t \left[\Lambda_{2t+1} q^b_{2t+1} r_{2t+1} + m_{2t+1}(1-\delta) \right], \tag{25}$$

$$m_{1t} = \beta E_t \left[\Lambda_{1t+1} q^a_{1t+1} r_{1t+1} + m_{1t+1}(1-\delta) \right], \tag{26}$$

where

$$m_{jt} = \Lambda_{jt}, \text{ for } j = 1, 2, \tag{27}$$

(x) Equations of moton for capital

$$k_{jt} = (1-\delta)k_{jt-1} + i_{jt}, \text{ for } j = 1, 2. \tag{28}$$

A.1.3 Model with Capital Adjustment Cost

As well as conditions (i)-(viii), the optimality conditions of models with capital adjustment costs include:

(ix) Euler equations

$$m_{1t} = \beta E_t \left[\Lambda_{1t+1} q^a_{1t+1} r_{1t+1} + m_{1t+1} \left(1 - \delta - \delta^{1/\xi} \left(\frac{1}{1-\xi} \right) \left(\frac{i_{1t}}{k_{1t-1}} \right)^{1-1/\xi} + \frac{\delta}{1-\xi} \right) \right], \tag{29}$$

$$m_{2t} = \beta E_t \left[\Lambda_{2t+1} q^b_{2t+1} r_{2t+1} + m_{2t+1} \left(1 - \delta - \delta^{1/\xi} \left(\frac{1}{1-\xi} \right) \left(\frac{i_{2t}}{k_{2t-1}} \right)^{1-1/\xi} + \frac{\delta}{1-\xi} \right) \right], \tag{30}$$

where

$$m_{jt} \delta^{1/\xi} \left(\frac{i_{jt}}{k_{jt-1}} \right)^{-1/\xi} = \Lambda_{1t}, \text{ for } j = 1, 2. \tag{31}$$

(x) Equations of moton for capital

$$k_{jt} = (1-\delta)k_{jt-1} + \left(\frac{\delta^{1/\xi}}{1-1/\xi} \left(\frac{i_{jt}}{k_{jt-1}} \right)^{1-1/\xi} + \frac{\delta}{1-\xi} \right) k_{jt-1}, \text{ for } j = 1, 2. \tag{32}$$

A.1.4 Model with Investment Adjustment Costs

In addition to the common conditions (i)-(viii), the optimality conditions of the model with investment adjustment costs include:

(ix) Euler equations

$$m_{1t} = \beta E_t \left[\Lambda_{1t+1} q^a_{1t+1} r_{1t+1} + m_{1t+1}(1-\delta)\right], \tag{33}$$

$$m_{2t} = \beta E_t \left[\Lambda_{2t+1} q^b_{2t+1} r_{2t+1} + m_{2t+1}(1-\delta)\right], \tag{34}$$

where

$$\Lambda_{1t} = m_{1t}\left[1 - \frac{\chi}{2}\left(\frac{i_{1t}}{i_{1t-1}} - 1\right)^2 - \chi\left(\frac{i_{1t}}{i_{1t-1}} - 1\right)\frac{i_{1t}}{i_{1t-1}}\right]$$
$$+\beta E_t\left[m_{1t+1}\chi\left(\frac{i_{1t+1}}{i_{1t}} - 1\right)\left(\frac{i_{1t+1}}{i_{1t}}\right)^2\right], \text{ for } j = 1, 2, \tag{35}$$

(x) Equations of moton for capital

$$k_{1t} = (1-\delta)k_{1t-1} + \left[1 - \frac{\chi}{2}\left(\frac{i_{1t}}{i_{1t-1}} - 1\right)^2\right] i_{1t}, \text{ for } j = 1, 2. \tag{36}$$

A.1.5 Common Optimality Conditions: Incomplete Markets

Depending on the adjustment costs formulation, the same conditions (ix)-(x) as the complete market models characterize the incomplete market models. Most of the common conditions described in section A.1.1 (i)-(vii) operate in incomplete markets. The only exception is the risk-sharing condition (viii), which is replaced with the following three conditions:

(viii) Risk-sharing condition

$$E_t\left[\beta\frac{\Lambda_{2t+1}}{\Lambda_{2t}}\frac{q^a_{1t+1}}{q^a_{1t}}\frac{RER_t}{RER_{t+1}}\right] = E_t\left[\beta\frac{\Lambda_{1t+1}}{\Lambda_{1t}}\frac{q^a_{1t+1}}{q^a_{1t}}\right] - \phi B_{1t}, \tag{37}$$

(xi) Bond-pricing condition

$$Q_t = E_t\left[\beta\frac{\Lambda_{1t+1}}{\Lambda_{1t}}\frac{q^a_{1t+1}}{q^a_{1t}}\right] - \phi B_{1t}, \tag{38}$$

(xii) Equation of motion for bonds

$$q^a_{1t} Q_t B_{1t} = q^a_{1t} a_{2t} - RER_t q^b_{2t} b_{1t} + q^a_{1t} B_{1t-1} - q^a_{1t}\phi B_{1t}. \tag{39}$$

A.2 Calibrated Parameters and Steady-State Values

This section describes the process of mapping the calibration targets to the models parameters. To facilitate replication of the results, the steady-state values of the endogenous variables are also reported. As described in Section 3.5, the steady-state calibration targets are as follows:

(T1) Hours worked as a fraction of time endowment: $n = 1/3$;

(T2) Investment-to-GDP ratio: $i/(qy) = 0.25$;

(T3) Capital-to-GDP ratio: $k/(qy) = 10$;

(T4) Steady-state import share: $im = b_1/y_1 = a_2/y_2 = 0.15$.

Several parameters common to the literature are also used: $\alpha = 0.36; \sigma = 2; \eta = 1/1.5$. The quarterly depreciation rate is in accordance with the calibration targets (T2) and (T3):

$$\delta = i/k = (i/qy)/(k/qy) = 0.025.$$

The discount factor is calculated using the values for α, δ and the steady-state capital output ratio (T3):

$$\beta = \frac{1}{\alpha\left(qy/k\right) + 1 - \delta} = 0.9891.$$

Since $b_1 = a_2$, steady-state import share im implies that

$$\frac{a_1}{b_1} = \frac{y_1 - a_2}{b_1} = \frac{(1 - b_1/y_1)}{b_1/y_1} = \frac{(1 - im)}{im} = 5.6667.$$

In the steady state, terms of trade, TOT, is expressed as

$$TOT = \frac{q_1^b}{q_1^a} = \frac{1-\omega}{\omega}\left(\frac{a_1}{b_1}\right)^\eta = 1,$$

from which it follows that the home bias parameter, ω, is

$$\omega = \frac{(a_1/b_1)^\eta}{1 + (a_1/b_1)^\eta} = 0.7607.$$

Because $TOT = 1$, all the intermediate good prices are the same. The steady-state price of any intermediate good, therefore, is given by

$$q = \omega\left(\omega + (1-\omega)\left(b_1/a_1\right)^{(1-\eta)}\right)^{\frac{1}{1-\eta} - 1} = 0.6092.$$

Determined by the targets (T1) and (T3) and the price of intermediate good, the steady-state capital is

$$k = \left[(k/qy) \cdot qn^{1-\alpha}\right]^{1/(1-\alpha)} = 5.6114.$$

Investment is expressed as

$$i = \delta k = 0.1403,$$

output measured in domestically produced goods as

$$y = k^{\alpha} n^{1-\alpha} = 0.9211.$$

and the steady-state use of intermediate goods as

$$a_1 = b_2 = (1 - im) \cdot y = 0.7829,$$

and

$$a_2 = b_1 = im \cdot y = 0.1382.$$

In the steady state net exports are zero, and GDP equals domestic absorption:

$$qy = i + c.$$

Consumption, therefore, is expressed as

$$c = qy - i = 0.4209.$$

From the labor supply equation

$$\frac{\gamma}{(1-\gamma)} \frac{c}{(1-n)} = qw,$$

and using the calibration target (T4) we derive the weight of labor, γ, in the utility function:

$$\gamma = \frac{qw(1-n)/c}{1 + qw(1-n)/c} = 0.6305.$$

Finally, the parameters ξ and χ that govern the magnitude of adjustment costs are set in order to ensure that volatility of investment relative to volatility of output matches the data. In our baseline model, CAC parameter is $\xi = 29$ and IAC parameter is $\chi = 0.093$.

Table 2: Business Cycle Statistics

	Data US-EU15 (1973:1 2012:2)	Predictions of the Model with						
		Complete Markets					Incomplete Markets	
		Standard Preferences			Time Nonseparable GHH Preferences		Standard Preferences	
		No AC	CAC	IAC	CAC	IAC	CAC	IAC
Panel A - Volatilities - Standard deviation (in %)								
Output	1.34	1.38	1.30	1.28	1.66	1.60	1.18	1.17
Net Export/Output	0.49	0.29	0.14	0.16	0.04	0.19	0.18	0.18
Terms of Trade	2.12	0.48	0.65	0.68	0.90	1.00	0.40	0.45
Real Exchange Rate	3.67	0.34	0.45	0.47	0.63	0.70	0.28	0.32
Real Interest Rate	1.25	0.04	0.03	0.11	0.05	0.59	0.03	0.14
Standard deviations relative to output								
Consumption	0.77	0.50	0.55	0.57	0.62	0.63	0.60	0.60
Investment	2.81	3.42	2.81	2.81	2.81	2.81	2.81	2.81
Employment	0.77	0.39	0.34	0.33	0.59	0.60	0.19	0.19
Capital	0.50	0.26	0.22	0.25	0.20	0.23	0.31	0.24
Panel B - Correlations with output								
Consumption	0.84	0.91	0.95	0.94	0.80	0.80	0.96	0.95
Investment	0.93	0.93	0.95	0.94	0.91	0.91	0.93	0.93
Employment	0.84	0.93	0.94	0.93	0.99	0.96	0.92	0.92
Capital	0.05	0.05	0.04	−0.01	0.26	0.26	0.31	0.26
Net Exports/Output	−0.44	−0.60	−0.53	−0.47	−0.39	−0.10	−0.59	−0.54
Real Interest Rate	0.27	0.95	0.50	0.36	0.23	0.35	0.49	−0.31
Panel C - Cross country correlations								
Output	0.62	0.17	0.28	0.29	0.57	0.58	0.29	0.30
Consumption	0.46	0.82	0.77	0.77	0.86	0.86	0.74	0.74
Investment	0.54	−0.54	−0.30	−0.32	0.35	0.35	−0.41	−0.39
Employment	0.55	−0.57	−0.36	−0.38	0.63	0.58	−0.44	−0.41
Panel D - Autocorrelations								
Output	0.89	0.69	0.70	0.74	0.73	0.82	0.70	0.72
Consumption	0.89	0.74	0.73	0.66	0.93	0.94	0.73	0.67
Investment	0.91	0.65	0.66	0.82	0.61	0.77	0.66	0.78
Employment	0.91	0.66	0.66	0.80	0.75	0.88	0.66	0.78
Capital	0.97	0.95	0.95	0.96	0.94	0.95	0.95	0.96
Net Exports/Output	0.79	0.67	0.65	0.85	0.91	0.49	0.60	0.84
Panel E - Correlations with investment								
Saving	0.86	0.99	0.99	0.99	1.00	0.99	0.99	0.99

Note: The statistics in the Data row are calculated from U.S. data and aggregated data for 15 European countries. The sample consists of the quarterly time series covering the period of 1973:1-2012:2. The model's statistics are computed from a single simulation of 100,000 periods. All the statistics are based on logged (except for the net exports) and HP-filtered data with a smoothing parameter of 1600.